



# Understanding a PD facilitator's decision-making and practices: prioritizing productive professional relationships to meet teachers' PD needs

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## Abstract

This study demonstrates the complexity of the decision-making process and the resultant practices of a PD facilitator to support two teachers to use an instructional protocol in mathematics. Through the use of the Resources, Orientations, Goals, and Identities (ROGI) framework, the role that my goals and identities played in the decision-making process and practices that followed are examined after transitioning into the role of instructional coach. The use of the ROGI framework also led to insights about how the two teachers' orientations and identities in particular influenced my decisions and practices. Using a self-study methodological frame, it is shown why the decision was made to prioritize co-teaching with one teacher in order to support her professional development. In the case of the second teacher, my early decisions about how to support her instruction did not lead to practices that supported her actual PD needs. Looking across the two cases, co-teaching was an effective strategy for building productive professional relationships with both teachers. Through cultivating such relationships, particularly by being vulnerable, it is shown that PD facilitators can gain insights that inform them about how to meet the PD needs of practicing teachers. This study contributes to the emerging research literature on the decision-making processes and practices of PD facilitators in mathematics. Among this study's unique contributions are findings about the potential consequences of a PD facilitator building productive professional relationships with teachers.

**Keywords** Professional development of mathematics teachers · Facilitators' decision-making · Facilitators' practices · Instructional coaching in mathematics · Productive professional relationships

During my career as a Mathematics Teacher Educator (MTE), I have prepared prospective teachers of mathematics (Rodríguez & Kitchen, 2005) and worked as a professional development (PD) facilitator for practicing teachers of mathematics (Kitchen et al., 2021).

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PD facilitators<sup>1</sup> “manage the PD activities, set norms for interactions, support teachers’ exchange of experiences and insights, monitor the discussion, and work with teachers toward the goals set for the PD” (Karsenty et al., 2023, p. 28). Similar to other PD facilitators, I believe that this work can lead to improvements in my participants’ instruction (Kennedy, 2016). PD facilitators wear many hats; in addition to leading professional development in mathematics for cohorts of teachers, PD facilitators may work as instructional coaches with PD participants in their classrooms (Kitchen et al., 2021). However, research on how PD facilitators make decisions about their practices is under-investigated (Karsenty et al., 2023; Kuzle & Biehler, 2015).

The focus of this paper is on my decision-making process and the practices I pursued as a PD facilitator to support two teachers to use an instructional protocol in mathematics called the Discursive Mathematics Protocol (DMP). Through the use of the Resources, Orientations, Goals, and Identities (ROGI) framework (Karsenty et al., 2023; Schoenfeld, 2010), the role that my goals and identities played in the decision-making process in particular and the resultant practices are examined after transitioning into the role of instructional coach. The use of the ROGI framework also led to insights about how the two PD participants’ resources, orientations, and identities influenced my decisions and practices. The following research question is addressed in this study:

How did I derive the decisions that I made as a PD facilitator in mathematics and what subsequent practices were carried out to support teachers to use an instructional protocol?

This study contributes to the research literature by providing insights about the decision-making process that a PD facilitator pursued and the subsequent practices that the facilitator engaged in after transitioning to the role of instructional coach. Another contribution of this study is that it demonstrates the potential value of cultivating productive professional relationships with PD participants as a means to meet their PD needs. In the ROGI analysis section, information is given about developing productive professional relationships with PD participants, one of my primary goals as a PD facilitator. Fostering productive professional relationships entails promoting trust among participating teachers as a means to provide supports teachers need to improve instruction (Kastberg & Kitchen, 2024; Sztajn et al., 2007). Prior to presenting the research findings, reviews of relevant research literature on professional development in mathematics and PD facilitators are provided. The ROGI framework, the theoretical framework used in this study, is also introduced. The research context and methods used in the study will also be presented.

## Professional development in mathematics

Teachers need access to quality (PD) opportunities to acquire knowledge and skills required to improve their students’ learning and that promote dynamic pedagogical approaches that actively engage students in learning mathematics such as problem-solving-based instruction (Fishman et al., 2003; Loucks-Horsley et al., 2010; Sowder, 2007; Sztajn et al., 2007). Importantly, when teachers can immediately implement in their classrooms what they learned in PD, the knowledge teachers have gained through

<sup>1</sup> A variety of terms that are synonyms with PD facilitators have been used in the research literature, such as PD providers (Even, 2005), leaders (Borko et al., 2014), and mentor-teachers (Kuzle & Biehler, 2015).

the PD is more sustainable (Liu & Phelps, 2020). However, the effects of PD may not last long. Knowledge loss is more acute for teachers who participate in summer PD programs than for those who participate in PD programs that are embedded in the school year. Liu and Phelps (2020) found that teachers' knowledge loss in content knowledge for teaching mathematics (CKT-M) can be significant; teachers lose much of what they learned in CKT-M PD in just over one month. An implication is that PD programs that take place over a longer duration (Garet et al., 2001), that take place during the school year, and that include more contact hours (Liu & Phelps, 2020) lead to sustained participant learning. Moreover, among the most salient features of PD that lead to improvements in participants' teaching practices is providing teachers with ongoing supports outside of the PD trainings (Copur-Gencturk & Papkonstantinou, 2016). Such supports can include observing teachers' lessons and giving them feedback on their instruction.

Instructional coaching has received attention in the research literature because of its potential to provide individualized PD supports to teachers to improve their instruction at the classroom level (Borko, 2004; Campbell & Malkus, 2011; Knight, 2007). For the purposes of this paper, an instructional coach is defined as someone who engages in school-based, job-embedded professional development (Polly et al., 2015). Cobb and Jackson (2011) included instructional coaching as one of their primary pillars in their theory of action to improve teaching at scale. Campbell and Malkus (2011) found that the mathematics achievement increased of students of elementary school teachers who had sustained classroom-supports from mathematics instructional coaches over time. Kraft and Blazar (2018) found that instructional coaching can significantly improve the quality of teachers' instruction, while positively impacting their students' achievement.

In the PD offered on the Discursive Mathematics Protocol, significant supports are given to teachers through the academic year to implement the DMP after they receive the initial training on the protocol. By providing these supports for a sustained period of time over the academic year and in a manner that is ongoing (Copur-Gencturk & Papkonstantinou, 2016; Garet et al., 2001; Liu & Phelps, 2020), the goal is to sustain participants' understanding of the DMP and how to use it, while galvanizing them to use the protocol in ways that lead to increases in their students' learning. The classroom-level supports given involve the PD facilitator moving into the role of an instructional coach who collaborates with PD participants to both plan and implement problem-solving-based lessons that are guided by the DMP.

## Research on PD facilitators

While there has been research on the decision-making and practices of PD facilitators of mathematics (Coles, 2019; Karsenty et al., 2023; Kuzle & Biehler, 2015), more research has focused on the skills and knowledge they need to carry out their work (Elliott et al., 2009; Zaslavsky & Leikin, 2004) and on their preparation and development (Rösken-Winter, 2015; Tzur, 2001). Elliott et al. (2009) examined the skills and knowledge that PD facilitators need to foster mathematically rich PD. Among their findings, Elliott et al. (2009) found that the skills and knowledge needed by PD facilitators were underspecified. Based on this finding, they refined a framework for the skills and knowledge needed by PD facilitators. This framework specifically attends to the Specialized Content Knowledge needed by teachers to effectively teach mathematics (Ball et al., 2008).

Zaslavsky and Leikin (2004) advanced a conceptual model to understand PD facilitators' professional growth. This model was proposed with the understanding that the development of PD facilitators' skills and knowledge is contingent upon the community of practice in which they engage.

In their study, Rösken-Winter et al. (2015) identified design principles to be used to prepare PD facilitators of mathematics for their work. Two design principles, competence-orientation and participant-orientation, are articulated as particularly important in the preparation of PD facilitators of mathematics. A competence-orientation is described as PD facilitators being well-prepared both in terms of their mathematics preparation and as mathematics educators, while a participant-orientation acknowledges the need for PD facilitators to understand the content and pedagogical needs of their participants (Rösken-Winter et al., 2015). Tzur (2001) reflected on his own development as a PD facilitator to generate a four-stage framework to characterize that development. The framework addresses how students learn mathematics, how PD facilitators support student learning, how PD facilitators develop ways of intentionally participating in their participants' learning, and how PD facilitators intentionally educate teachers of mathematics.

Similar to Coles (2019), Karsenty et al. (2023), and Kuzle and Biehler (2015), this study examines the decision-making process and practices of a PD facilitator. Coles (2019) reported on how a PD facilitator in mathematics PD makes decisions about participants' comments and how to respond to those comments. Specifically, in the PD described, the facilitator is orchestrating discussions that take place in video clubs. Much of the focus is on practices that the PD facilitator instigates based upon comments made during the video club discussions (Coles, 2019). A key finding of this study is that the PD facilitator's decision-making and practices are influenced by the "kinds" of comments that participating teachers made during video club meetings. In their study, Kuzle and Biehler (2015) reported on the practices of 12 PD facilitators who developed courses for practicing teachers in statistics. Among their findings was that there was great variation in terms of the PD facilitators' practices. The facilitators tended to draw largely on their own teaching experiences rather than on the research literature on effective approaches to instruction. As a result, the researchers plan to design trainings for the PD facilitators to better integrate theory, teaching, and practice (Kuzle & Biehler, 2015). Karsenty et al. (2023) used the ROGI framework that had previously been used to examine teachers' decision-making processes and practices as a means to understand two novice PD facilitators' decision-making processes and practices. Josh, one of the PD facilitators in the study, had a fragile identity as a facilitator that led him to rely heavily upon his dominant identities. Karsenty et al. (2023) showed that many of Josh's decisions and practices were guided by his teaching identity, something Josh regretted because of his desire to take on the stance and practices of a facilitator. On the other hand, Nina, another novice PD facilitator in the study, was able to move easily between her multiple identities as a teacher, mathematics department chair, and facilitator to support PD participants' learning. The reason for this was Nina's comfort with her multiple identities, making it easy for her to move between her many roles (Karsenty et al., 2023). A unique contribution of this study is that the decision-making process and practices of the PD facilitator are considered from the facilitator's perspective.

## Theoretical framework

Similar to Karsenty et al. (2023), the ROGI framework is used in this study to examine the decision-making and practices of a PD facilitator to support two PD participants' implementation of the DMP. A reason the ROGI framework was chosen for use in this study was because of the epistemological assumption embedded within the framework that decision-making precedes practice. It was also chosen because of the strong role that identity plays in the framework. My identity as a PD facilitator played a pivotal role in the decision-making process and the practices that I pursued with the two PD participants. Moreover, the ROGI framework is used to help understand the two PD participants' resources, orientations, and goals as well as aspects of their identities. As will be shown, my decision-making process and the resulting practices that ensued were influenced by the participants' orientations and identities.

The ROG framework was designed to understand how teachers make decisions when teaching (Schoenfeld, 2010) and consists of: (1) The various intellectual, material, contextual, and social resources (R) that a teacher draws on, (2) The orientations (O), beliefs, values, preferences that guide a teacher, and (3) The explicit and implicit goals (G) that drive a teacher. Karsenty et al. (2023) modified Schoenfeld's (2010) ROG framework by including Identity (I) and used the ROGI framework to understand how PD facilitators make decisions. Research on mathematical identity has historically involved studying student affect, such as interest in and enthusiasm for mathematics (Cobb et al., 2009). Recently, the notion of mathematical identity has been expanded to take into account one's "lived and negotiated experience with a particular culture" (Chval et al., 2010, p. 193) such as how teachers' involvement in professional development in which their ideas were continually leveraged to further other participants' mathematical thinking affected their identities (Kitchen, 2022; Kitchen et al., 2021). Identity, specifically how I identified with two PD participants, played a central role in this self-study of my decision-making process and practices as a PD facilitator.

Schoenfeld (2010) originally used the ROG to make sense of teachers' decision-making processes, so differences exist regarding how the three constructs of the ROG framework apply to PD facilitators (Karsenty et al. (2023)). For example, resources for PD facilitators include not only knowledge resources such as specialized mathematical content knowledge (Ball et al., 2008; Hill et al., 2005), but also knowledge concerning how teachers learn (Borko et al., 2014; Zaslavsky & Leikin, 2004) and knowledge about teachers' racialized and classed experiences as pre-tertiary students (Kitchen et al., 2021). Resources for PD facilitators can also include experiences that can help inform one's decisions and practices. For example, a PD facilitator could have significant experience collaborating with teachers in their classrooms, which can help inform the facilitator about how to meet teachers' PD needs (Campbell & Malkus, 2011). In addition to understanding the orientations that teachers have toward student learning, PD facilitators' orientations should include their views of teachers and notions about teacher learning (Karsenty et al., 2023). For example, a PD facilitator could view teachers from a deficit lens or, alternatively, as possessing wisdom of practice (Shulman, 2004). The content and process goals of PD facilitators have commonalities and differences to the content and process goals of teachers (Karsenty et al., 2023). Similar to teachers, PD facilitators focus extensively on the content they teach and how they teach it. However, PD facilitators may also include "atmosphere goals" (Karsenty et al., 2023, p. 30), such as creating a safe space for teachers to explore inquiry-based instruction with colleagues. In my work as a PD facilitator, a process goal is to develop

robust professional relationships with PD participants as a means to cultivate meaningful collaborations with them.

## Research context and methodology

My decisions and practices as a PD facilitator were investigated with two elementary school teachers who were implementing the Discursive Mathematics Protocol (DMP) during five problem-solving lessons throughout the 2022–2023 academic year. The DMP builds on George Pólya's (1945/1986) iconic problem solving heuristic by incorporating research-based “language practices” (LPs) grounded in theories of academic language development and acquisition (Erath et al., 2021) and “essential teaching practices” (ETPs) (National Council of Teachers of Mathematics [NCTM], 2014). The DMP is intended to be used as a guide by teachers during problem-solving lessons to support students to simultaneously learn the language of mathematics and develop their mathematical reasoning through the implementation of the LPs and ETPs. Additional information about the DMP can be found in Kitchen et al. (2024). Initially, the two teachers were introduced to the pedagogical principles underlying the DMP and how to use the protocol during problem-solving-based instruction in one full-day training. After this training, I shifted into the role of an instructional coach and me and others collaborated with the two teachers during the problem-solving lessons to support their implementation of the DMP. Lesson planning support was also provided to the participating teachers for every task prior to lesson implementation.

## Self-Study as methodological frame

To examine my decisions and practices as an instructional coach, I engaged in self-study. The goal of self-study is for researchers to systematically investigate their own perspectives and practices within their unique professional contexts (Borko et al., 2007; Loughran, 2007). As a form of practitioner research, self-study research uses qualitative methodologies to collect data generated in practice that represent “experiences of practice” (Kastberg et al., 2020, p. 137). Self-study researchers position themselves both as educational researchers and the researched (Pinnegar et al., 2020). From its inception, researchers have used self-study to improve their own professional practice (Loughran & Northfield, 1998). For example, Borowski and Rupe (2023) used self-study to examine the impact of collaborative planning mathematics courses for preservice teachers that they both taught. Lischka et al. (2023) engaged in a self-study to understand the progression of their use of alternative assessment practices in a mathematics methods course.

Self-study also has a long history of use by teacher educators for their own professional learning (Lunenberg et al., 2010; Vanassche & Kelchtermans, 2016). PD facilitators continually engage in self-study to make decisions about the practices that they pursue in PD trainings they lead (Kastberg & Kitchen, 2024). In this self-study, I intentionally sought to understand the decisions that I made and the resultant practices that I pursued after putting on the hat of instructional coach with the two participating teachers who were implementing the DMP. I now offer some information about myself as the sole participant in this study and some context for my work as a PD facilitator.

## Study participant

The author is the principal study participant. I have worked as a PD facilitator in mathematics for more than three decades. While the majority of this work has been with pre-tertiary teachers across all the grade levels, I have also facilitated PD sessions for instructors of undergraduate-level mathematics courses. I met Ms. Diaz, one of the two PD participants featured here, through my work as an instructor of Los Alamos National Laboratory's sponsored Math and Science Academy (MSA) "Institute." Through the Institute, I developed an instructional approach referred to as leveraging teachers' mathematical ideas (LTMI). LTMI was designed with three goals in mind: (1) To enhance teachers' mathematical knowledge, (2) To make teachers' mathematical ideas a central focus of PD, and (3) To moderate teachers' unproductive beliefs about themselves and their students such as deficit beliefs they may hold about their own mathematical abilities (Kitchen, 2022). The latter two aspects of the LTMI were included because the vast majority of Institute participants are minoritized women who attended schools in northern New Mexico where they had experienced an impoverished mathematics education as pre-tertiary students (Kitchen, 2022). One of the main goals of the LTMI was to empower Institute participants through collectively nurturing the understanding that they had insightful mathematical ideas that others could learn from. Kitchen et al. (2021) found that through their engagement in the Institute, participants' mathematical identities were positively transformed.

The vast majority of my work as a PD facilitator has been carried out in the state of New Mexico, USA. Working in New Mexico has profoundly influenced my identity as a PD facilitator. New Mexico is culturally unique in the US context, given that significant percentages of its population are Hispanic (50.2%) and American Indian (11.2%) (U.S. Census Bureau, 2022). Similar to only seven other US states, New Mexico is a "majority minority" state, meaning that a majority of its population identifies as non-white (Majority minority in the United States, n.d.). The ethnic diversity of New Mexico is also reflected among its teachers, almost 39.2% of the state's teachers are Hispanic or American Indian (National Center for Education Statistics, 2018). This contrasts sharply with the national demographics of US teachers; only about 10% of all pre-tertiary teachers in the USA identify as Hispanic or American Indian (National Center for Education Statistics, 2023). The vast majority of teachers with whom I have worked as a PD facilitator in New Mexico are minoritized women who grew up and attended schools in their home communities in New Mexico (Kitchen, 2022). In the US context, minoritized people are those of diverse ethnic and racial backgrounds. Through my work in New Mexico over the years, I have come to identify strongly with its teachers, specifically minoritized women who are passionate about and committed to serving their students, mainly minoritized students from economically marginalized communities.

## PD participants

The two participating teachers were "Ms. Diaz," a 6th-grade teacher who worked in a highly diverse elementary school situated in a rural community in northern New Mexico and "Ms. Baker," a 4th-grade teacher who worked in a highly diverse elementary school located in an urban community in Colorado. "Ms. Diaz" had been a participant in the Institute. Ms. Baker had been a student of mine when she was completing her elementary school teaching certification. Both worked at schools that were largely populated by minoritized students. Many minoritized students speak a language at home that differs from the



**Table 1** Information about the PD participants

| Pseudonym | Gender identity | Ethnicity or race | Grade taught | Years teaching | Highest degree obtained |
|-----------|-----------------|-------------------|--------------|----------------|-------------------------|
| Ms. Diaz  | Female          | Hispanic          | 6th grade    | 22 years       | MA in Education         |
| Ms. Baker | Female          | White             | 4th grade    | 6 years        | MA in Education         |

language of instruction, which is English in US schools. Ms. Diaz' school was attended primarily by Hispanic<sup>2</sup> and American Indian students from local Pueblos.<sup>3</sup> Ms. Baker's school was largely attended by immigrant children, the majority of whom identified as Mexican.

I asked Ms. Diaz if she would be willing to be part of a study of the DMP so that I could learn from her about how to improve the instructional protocol. Two MSA personnel, Drs. Lorenzo Gonzales and Karla Matute (KM), accompanied me to Ms. Diaz' class when she implemented each of the five problem-solving lessons (throughout, I refer to the three of us as "the team"). It is noteworthy that though I had been Ms. Diaz' instructor in the Institute and Dr. Gonzales<sup>4</sup> had been her teacher during her teacher education program, Ms. Diaz was not intimidated by having the two of us and KM observe her teach the problem-solving lessons. Ms. Diaz made a point early on to emphasize to us that she was happy to have us visit her classroom and to contribute to her learning and her students' learning. Ms. Baker invited me to both teach her about the DMP and to support her implementing the DMP in her class. In the fall of 2022, a graduate assistant (GA) accompanied me to observe Ms. Baker teach. Similar to what occurred in Ms. Diaz' case, the GA and me circulated with Ms. Baker throughout the classroom during group work to ask individual and groups of students questions. We also engaged students in mathematical discourse. In the spring of 2023, I did not have a GA to assist me during visits to Ms. Baker's classroom. In terms of the power dynamic that existed between us, I was no longer Ms. Baker's professor when we initiated our collaboration in the fall of 2022. Our relationship was based simply on Ms. Baker's desire to learn about the DMP.

Information about the PD participants is given in Table 1. Ms. Diaz identifies as a Hispanic woman and Ms. Baker identifies as a white woman. As previously indicated, both of the participants taught at the elementary school level, at schools located in diverse, economically marginalized communities. Each of the participants had completed an undergraduate degree in elementary education and both held a master's degree in Education. Ms. Diaz was a veteran teacher who had taught for 22 years and Ms. Baker was in her 7th year as a teacher when the study was undertaken.

## Data collection

Throughout the 2022–2023 school year, the data drawn upon in this self-study are from classroom observations made in the PD participants' classrooms when they implemented

<sup>2</sup> "Hispanic" is a term commonly used in New Mexico and denotes people of Spanish descent.

<sup>3</sup> New Mexico's eight northern Pueblos include: Nambé, Ohkay Owingeh (formerly San Juan), Picuris, Pojoaque, San Ildefonso, Santa Clara, Taos, and Tesuque.

<sup>4</sup> Dr. Gonzales died in July, 2023. This article is dedicated to him. A twenty-first generation New Mexican, Dr. Gonzales devoted his life to improving mathematics and science education in northern New Mexico.



each of the five problem-solving lessons, from conversations that took place with the PD participants during lesson planning sessions and the actual lessons, from interviews that were conducted with the two participants, and from videotapes made when the PD participants carried out the problem-solving lessons with their students. Notes were taken about both the observations made during the actual lessons and the conversations that took place with the participants, particularly conversations that occurred during lesson planning sessions and the actual lessons. I kept a journal of these notes to keep track of key ideas and insights gleaned from these observations and conversations. Many of the insights included in the ROGI analyses were compiled from the classroom observations made and the conversations that occurred. For example, the teachers' visible practices were identified from classroom observations made. I learned of Ms. Diaz' strong identification with her students from several conversations that took place, often during actual problem-solving lessons. I also came to understand the importance that Ms. Baker placed on teaching using equitable instructional strategies from conversations that we had.

Two individual interviews were conducted with each of the two participating teachers (4 total), one in November 2022 and the other in May of 2023. Each of these semi-structured interviews was carried out via Zoom. Each of the four interviews was recorded and lasted between 60 and 75 min. Five questions were asked in the first interview that took place, including one with a predetermined probe question. Six interview questions were asked in the second interview, two with a predetermined probe question (see Appendix A for interview questions asked during the interviews). The goal of the interviews was to engage participants in a conversation through the use of open-ended questions that prompted the teachers "to construct answers" (Riessman, 1993, p. 54) in collaboration with the researcher/interviewer. The questions asked focused largely on the DMP, participants' experiences implementing the instructional protocol, and their ideas were solicited about how the DMP could be improved. As will be shown, some of what was learned from these interviews informed the ROGI analyses completed for the PD participants, particularly their identities and orientations.

Videotapes of each of the PD participants' problem-solving lessons were created and transcribed to help document and reconstruct what occurred during the lessons. Students who had provided consent along with their parents were placed in the same group or groups to make it easier to videotape these students only and so that students who did not provide consent were not videotaped. To help further inform data analysis, the videotapes were used to reference aspects of the problem-solving lessons to gain insights from participating teachers. While the videotapes were made during problem-solving lessons, the team and I were actively observing and participating in the lessons as previously described. Review of the videotapes helped to substantiate observations made about the PD participants' "visible practices," such as the insight that Ms. Diaz consistently did not reference lesson plans during the problem-solving lessons.

## Data analysis

Understanding the decisions made and the practices that I ultimately pursued were derived through reflectively analyzing what transpired in my work with the teachers and then documenting in a journal what happened over the course of the year. An example was that I documented how significant time was devoted solely to passively observing Ms. Baker without doing much to assist her. This contrasted with the co-teaching that the team had initiated with Ms. Diaz. Analysis entailed not only examining interview data, videotapes,

and notes taken on conversations and observations made, but also appraising the events that occurred in my work with the teachers and how these events unfolded through the year. For example, considerable time was devoted to examining what I could do differently to more actively support Ms. Baker during the problem-solving lessons.

The notes about observations made and conversations that occurred between me and the PD participants as well as the interviews and videotapes were reviewed using a thematic analysis approach in which the focus is more on what is said rather than on how it is told (Riessman, 1993). An iterative process was used to examine themes that emerged across this analysis, reflecting upon, and then clarifying participating teachers' views (Miles et al., 2013). With a thematic analysis, the goal is to first collect narratives, then to inductively group similar narratives, and finally to select key narratives told and ideas conveyed for illustration purposes (Riessman, 1993). Among the most noteworthy theme to emerge across the conversations and the interviews conducted with the teachers concerned how they could have been better supported to implement the DMP as the year progressed. This finding helped me realize that some of my initial decisions and practices as a PD facilitator did not serve the teachers particularly well. Other themes that emerged included: (1) The conversations that took place with both teachers became more authentic as the year progressed, and (2) Both teachers wanted more of a focus on the language practices embedded in the DMP in our collaborative work. Themes that emerged that were specific to Ms. Diaz was that the PD provided on the DMP should have been more explicit and that she wanted to move away from focusing almost exclusively on student perseverance. For Ms. Baker, a theme that emerged was that she wanted more supports provided during the actual problem-solving lessons.

## Ethical Considerations

Ethical approval for this research project was granted through the University of Wyoming's Institutional Review Board (Protocol #: 20181004RK02141), and written consent to participate in this study was given by each PD participant. Pseudonyms are used for participants to maintain their anonymity. The author states that there is no conflict of interest.

## ROGI analysis of the PD facilitator

ROGI analyses are now done of the PD facilitator and the two PD participants. This ROGI analysis reflects my resources, orientations, content and process goals, and identity as a PD facilitator. Important resources for me as a PD facilitator include having extensive experience leading mathematics PD for pre-tertiary teachers, possessing a strong background in mathematical problem-solving and engaging teachers in problem-solving-based lessons, and having significant experience collaborating with teachers in their classrooms (Kitchen, 2022). As a PD facilitator, one of my primary orientations is that teachers have wisdom of practice (Shulman, 2004). Taking this wisdom as a given, I want to learn from teachers about their insights about how to improve the DMP. While a content goal is to support teachers to implement the DMP's components, I also want them to be empowered to experiment with DMP implementation. My identities as a PD facilitator include wanting to demonstrate that I am a partner who cares about teachers and the important work they do. I have insights about the DMP to offer teachers, but teachers are the experts when it comes

to their students and their specific instructional needs, and I identify strongly with teachers in the challenging work they do.

My primary process goal as a PD facilitator is to develop productive professional relationships<sup>5</sup> with teachers as a means to directly assist them in the challenging job of implementing the DMP. Cultivating productive professional relationships entails collaborating with teachers in a manner that explicitly conveys respect for the participants and their expertise as teachers (Kitchen, 2023). Analogously, it entails demonstrating through one's actions that the PD facilitator does not have all the answers. Productive professional relationships are nurtured through developing caring relationships with teachers during PD activities and leads to PD facilitators cultivating trust among the teachers they serve (Sztajn et al., 2007). A means to cultivate caring relationships is through engrossment, which entails listening to and attending to participants' needs (Noddings, 2002). As caring relationships develop, trust is built and teachers become more willing to share their teaching challenges (Battey & Chan, 2010). Bartell (2011), Battey and Chan (2010), Kastberg and Kitchen (2024), and Sztajn et al. (2007) provide explicit examples about the value of fostering productive professional relationships with teachers in their work as PD facilitators as a means to build trust with their PD participants. For instance, Battey and Chan (2010) explicitly established productive professional relationships with teachers in PD that they led as a means to open spaces for difficult discussions with their participants about instructional practices that they needed to consider implementing to support the achievement of their minoritized students.

Productive professional relationships were advanced with the two PD participants through initially identifying with them. I identified with the two PD participants in this study because I had worked extensively with both of them in the past. Identifying with the PD participants also meant that I believed in both teachers' abilities to teach mathematics well. I viewed both Ms. Diaz and Ms. Baker as competent teachers who could orchestrate challenging, problem-solving-based lessons. I also identified with the two PD participants because of their commitment to teach minoritized students who made up the majority of the students in their classrooms. Identifying with participants provides a means to humanize them and demonstrates a willingness to partner with them in the demanding work of mathematically educating their students (Kitchen, 2018). While identifying with the participants was important to cultivate productive professional relationships, it will be shown that being vulnerable with the participants, another potential feature of productive professional relationships (Lasky, 2005) proved essential to gain insights that eventually led to meeting the actual PD needs of the participants. Aspelin (2021) describes relational bonds that grow between teacher and student. The development of these bonds is contingent upon the teacher being professionally vulnerable by revealing their humanity to their students (Lasky, 2005).

Table 2 summarizes the ROGI analysis of the PD facilitator (myself). Following Karsenty et al. (2023), the PD facilitator's visible practices are specified first, followed by my underlying resources, declared process goals, enacted content goals, identities, and orientations.

<sup>5</sup> The terms "productive professional relationships" and "relational practices" are used synonymously in the research literature (Kastberg & Kitchen, 2024).

**Table 2** Summary of ROGI analysis of the PD Facilitator

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| <p>The PD facilitator's <b>visible</b> practices:</p> <ul style="list-style-type: none"> <li>• In PD sessions, works to introduce theory behind the DMP and then to clarify the theory with examples of practices that follow.</li> <li>• As an instructional coach, significant attention is paid to collaborating directly with teachers to support DMP implementation during problem-solving lessons. For example, the PD facilitator is actively engaging individual students and groups of students during lessons in mathematic discourse when appropriate (rather than simply passively observing the lesson).</li> <li>• A significant focus is placed on cultivating caring relationships through engrossment, which entails listening to and attending to participants' needs (Noddings, 2002).</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <p><b>Underlying</b></p> <p>PD facilitator's resources:</p> <ul style="list-style-type: none"> <li>• Strong background in mathematical problem-solving and in engaging teachers in problem-solving based lessons.</li> <li>• Extensive experience leading PD in mathematics for pre-tertiary teachers who teach at a variety of levels.</li> <li>• Extensive experience collaborating with teachers in their classrooms.</li> </ul> <p>Enacted content goal:</p> <ul style="list-style-type: none"> <li>• To support the teachers to implement the components of the DMP during problem-solving lessons.</li> <li>• There is no one right way to implement the instructional protocol. Want teachers to feel empowered to experiment with DMP implementation to simultaneously develop their students' mathematical reasoning and learning of the mathematics register.</li> </ul> <p>Declared process goals:</p> <ul style="list-style-type: none"> <li>• To engage in productive professional relationships with teachers as a means to honor and sustain them.</li> <li>• To collaborate with teachers in a respectful manner (i.e., in a manner that demonstrates that the PD facilitator does not have all the answers).</li> <li>• To support teachers directly in the classroom in the messy job of implementing a complex instructional protocol.</li> <li>• To validate teachers' instructional moves that prioritize student inquiry (e.g., posing purposeful questions and prompts).</li> </ul> <p>PD facilitator's identities:</p> <ul style="list-style-type: none"> <li>• I am here as a partner who cares about teachers and the important work they do with your students.</li> <li>• I have insights about the DMP to offer teachers, but teachers are the experts when it comes to their students and their specific instructional needs.</li> <li>• I identify strongly with teachers in the challenging work they do.</li> </ul> <p>Orientations:</p> <ul style="list-style-type: none"> <li>• Want to learn from teachers about the strengths and weaknesses of the DMP.</li> <li>• Believe that teachers have wisdom of practice (Shulman, 2004) and want to learn from teachers about how to improve the DMP.</li> </ul> |

## ROGI analysis of Ms. Diaz

From the outset of our collaboration with Ms. Diaz, it was clear from observations made

that she had solid classroom management skills that made it possible for her to implement problem-solving-based lessons. The implementation of problem-solving lessons is very demanding for teachers because these lessons require extensive planning, include intrinsic challenges such as maintaining the cognitive level of tasks used, and require skills to actively engage student participation (Silver & Smith, 1996). As a participant in the Institute, Ms. Diaz had developed a strong background in mathematical problem-solving; a significant emphasis is placed in these trainings on teachers engaging with their peers in problem-solving. Based on her experiences in the Institute, Ms. Diaz was well-informed about mathematical problem-solving. The team quickly learned from our classroom visits that Ms. Diaz was not hesitant to implement problem-solving lessons. We also observed that an important goal for her was to frequently review task instructions during the problem-solving lessons. During the lesson planning sessions that took place prior to each lesson, Ms. Diaz exuded confidence and seemed undeterred by the prospect of engaging her students in solving worthwhile tasks with our team present. Well-developed lesson plans were an outcome of these sessions. However, we observed that Ms. Diaz did not reference these plans during the problem-solving lessons. As a result, Ms. Diaz did not pose purposeful questions, nor did she intentionally implement other instructional practices that comprise the DMP as planned prior to the lessons. Rather, Ms. Diaz focused almost exclusively during the lessons on developing her students' perseverance to solve problems.

During the first lesson planning session, Ms. Diaz told team members about how she placed significant emphasis on her students engaging in "productive struggle" to solve problems. To her, this equated with allowing students extended periods of time to devise solution strategies and try out these strategies. During problem-solving lessons, we observed that Ms. Diaz generally avoided talking with her students or intervening in their work. Our team noticed that this was a pattern, after introducing the problem, Ms. Diaz would allow students to devise strategies to solve the problem for long periods of time without intervening. Ms. Diaz articulated to us that by not getting involved and giving her students long periods of time to devise strategies, she believed she was promoting her students' perseverance to problem-solve. However, the team observed students were often not talking about the problem under consideration as we circulated among them during these long periods of time. We observed significant student frustration as many struggled to progress to solve a problem, leading some students to give up on the problem. Rather than promoting perseverance, Ms. Diaz' primary instructional goal, students were becoming frustrated and often gave up on the task.

During interviews, Ms. Diaz frequently talked about how she was a veteran teacher who had experienced significant professional development throughout her career. She believed in herself as a teacher and she believed in her students. As a Hispanic woman, she identified strongly with her students, the majority of whom are also Hispanic, and they identified strongly with her. Though she was not from the town where she taught, she grew up near the town and identified deeply with her students. Because Ms. Diaz believed so strongly in her students, she made a point of praising her students often; praising them as people, but also as students who were capable and smart. Table 3 summarizes the ROGI analysis of Ms. Diaz.

**Table 3** Summary of ROGI analysis of Ms. Diaz

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| <p>Ms. Diaz' <b>visible</b> practices during problem-solving lessons:</p> <ul style="list-style-type: none"> <li>• Possessed solid classroom management skills that allowed for implementation of demanding problem-solving based lessons.</li> <li>• Was willing to implement problem-solving lessons.</li> <li>• Did not always follow lesson plan, though extensive lesson planning had occurred.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <p><b>Underlying</b></p> <p>Ms. Diaz' resources:</p> <ul style="list-style-type: none"> <li>• Strong background in mathematical problem-solving.</li> <li>• Promoted student "perseverance" by allowing them to struggle for long periods of time.</li> </ul> <p>Declared process goals:</p> <ul style="list-style-type: none"> <li>• Placed significant emphasis on student perseverance.</li> <li>• Allowed students to struggle during problem-solving lessons. Believed that through struggle, students would develop perseverance.</li> </ul> <p>Enacted content goal:</p> <ul style="list-style-type: none"> <li>• Frequently reviewed task instructions.</li> </ul> <p>Ms. Diaz' identities:</p> <ul style="list-style-type: none"> <li>• Proud that she was a veteran teacher.</li> <li>• Proud that she had experienced significant PD.</li> <li>• Identified strongly with her students and they identified strongly with her. Though she was not from the town where she taught, she grew up near the town and identified strongly with her students.</li> </ul> <p>Orientations:</p> <ul style="list-style-type: none"> <li>• Wanted to promote student perseverance.</li> <li>• Wanted to learn about language and how to promote her students' use of mathematical language.</li> </ul> |

### ROGI analysis of Ms. Baker

Similar to Ms. Diaz, Ms. Baker had strong classroom management skills that served her well when initiating and carrying out problem-solving-based lessons. Ms. Baker put these skills to use during lessons to orchestrate active student engagement during the lessons. Also similar to Ms. Diaz, she was not hesitant to implement problem-solving lessons. As part of her master's degree program in Education, Ms. Baker took courses with me as her instructor that were designed to support prospective teachers to learn specialized knowledge needed to teach elementary school mathematics. These courses included extensive experiences engaging in mathematical problem-solving as a learner with significant opportunities to reflect on these experiences and how to teach using problem-solving-based approaches. Because she had this background, Ms. Baker was confident teaching problem-solving lessons with her students.

In my initial observations of her instruction, Ms. Baker was quite capable of implementing the instructional practices included in the DMP. For example, she frequently elicited and used evidence of her students' thinking (ETP#3) during problem-solving lessons and a range of her students shared their ideas during whole class conversations. It was clear that Ms. Baker's students expected to be called upon during class discussions and, as a consequence, believed that their teacher appreciated the mathematical knowledge they brought to

the classroom (Kitchen et al., 2021). It was also the case that Ms. Baker employed strategies as suggested in the DMP at the beginning of lessons to help her students understand the task under consideration. For instance, Ms. Baker asked questions such as “What is the problem asking you to do?” and “How are you going to figure this out?” Moreover, Ms. Baker often focused on promoting her students’ mathematical reasoning and posing purposeful questions and prompts (ETP#2). Her students’ mathematical ideas were often a focal point of discourse that took place during instruction. Overall, she was quite competent at engaging her students in mathematical problem-solving and in implementing instructional practices such as those included in the DMP that promote mathematical reasoning.

During the Spring interview, Ms. Baker indicated that it was important to her to teach using equitable instructional strategies. She believed that every one of her students could learn mathematics. Ms. Baker wanted her minoritized students, in particular, to have frequent opportunities to interact during mathematics lessons in their primary language. In the Fall interview, Ms. Baker shared her belief that she was well-prepared to teach her minoritized students. She worked with a highly diverse group of students, and it was important to Ms. Baker to attend to their social, cultural, linguistic, and academic needs. One strategy that Ms. Baker explicitly used to support her minoritized students was to intentionally situate students who spoke Spanish as their primary language with other Spanish speakers during small group discussions. Table 4 summarizes the ROGI analysis of Ms. Baker.

## Decisions made and subsequent practices pursued to support the teachers

### Supporting Ms. Diaz: PD facilitator's response

Since Ms. Diaz was not implementing DMP instructional components in the problem-solving lessons and my declared process goal was to support teachers in the messy job of implementing a complex instruction protocol, the decision was made to proactively collaborate with Ms. Diaz to co-teach the problem-solving lessons as the year progressed. The primary goal of the co-teaching was to demonstrate how to implement DMP instructional components during problem-solving lessons. Co-teaching the lessons entailed both KM and me frequently advising Ms. Diaz about opportunities to implement DMP instructional practices at specific moments during the lessons and, at times, leading instruction. Through co-teaching with Ms. Diaz, our team demonstrated how to pose purposeful questions and engage students in mathematical discourse, both aspects of the DMP. Though perseverance is not an explicit component of the DMP, we elected to model how perseverance could be promoted during lessons because promoting student perseverance was one of Ms. Diaz’ orientations. Rather than allowing students to persevere with a problem for 25 min without any instructional support, our team modeled for Ms. Diaz how to solicit students’ ideas for discussion after allocating 5–10 min for students to generate ideas to solve problems. The idea was to have “check ins” in which students could explain and reflect upon their mathematical thinking, while engaging with their peers’ mathematical ideas (Schoenfeld, 1985; Webb et al., 2021).

**Reflecting on Ms. Diaz’ Case:** As the academic year progressed, conversations that took place between Ms. Diaz and team members, myself in particular, became more authentic. The more authentic discourse that eventually took place was made possible by an



**Table 4** Summary of ROGI analysis of Ms. Baker

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
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| <p>Ms. Baker's <b>visible</b> practices during problem-solving lessons:</p> <ul style="list-style-type: none"> <li>• Possessed solid classroom management skills that allowed for implementation of demanding problem-solving based lessons.</li> <li>• Was confident about implementing problem-solving lessons.</li> <li>• Focused extensively on developing her students' mathematical reasoning and posing purposeful questions.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <p><b>Underlying</b></p> <p>Ms. Baker's resources:</p> <ul style="list-style-type: none"> <li>• Strong background in mathematical problem-solving.</li> <li>• Good mathematics background.</li> <li>• Well-prepared to implement problem-solving based lessons.</li> </ul> <p>Declared process goals:</p> <ul style="list-style-type: none"> <li>• Engage a diversity of students in sharing their solutions.</li> </ul> <p>Enacted content goal:</p> <ul style="list-style-type: none"> <li>• Devoted time at beginning of lesson to ensure students understood the task under consideration.</li> </ul> <p>Ms. Baker's identities:</p> <ul style="list-style-type: none"> <li>• Believed that she was well-prepared to teach minoritized students and placed significant emphasis on teaching using equitable instructional strategies – believed that every one of her students could learn mathematics.</li> </ul> <p>Orientations:</p> <ul style="list-style-type: none"> <li>• Wanted to engage in collaborative teaching and decision-making during instruction of problem-solving lessons.</li> <li>• Wanted to learn about language and how to promote her students' use of mathematical language.</li> </ul> |

uncomfortable exchange that occurred at the conclusion of the fourth lesson when I conveyed to Ms. Diaz my belief that only a handful of her students had engaged in much depth in the lesson. This surprised and upset Ms. Diaz because her star student had beautifully explained his thinking near the conclusion of the lesson, which for Ms. Diaz, demonstrated that her students as a whole had done well solving the task. When we talked again several weeks later, I apologized for sharing my evaluation of the lesson given that we did not have time to unpack it in much depth because of time constraints. Soon thereafter, we began to engage in more authentic conversations during which Ms. Diaz was willing to listen to my perspectives on how she could more consistently implement the DMP during the problem-solving lessons. At that time, I also confessed to her my belief that I was not effectively mentoring and supporting her to implement the various components of the DMP during these lessons.

As evidence that our conversations had become more authentic as the year progressed, Ms. Diaz was quite forthright during the Spring interview about how the PD provided throughout the year could have more explicitly shown her what to focus on during DMP implementation: "It's just a process of figuring out what exactly was the goal I guess, and fulfilling that goal..." She shared that it would have helped her to have had "a list of things we can work on, maybe a concrete list like we'll work on this, and maybe even a checklist... I would have liked that a little sooner..." Ms. Diaz' blunt assessment of the PD supports she was given and not given and how she did not always understand "what exactly was the goal" of the PD points to the need to give clear guidance to teachers as

instructional coaches, particularly when working closely with them to implement a complex instructional protocol.

As another example of how our conversations became more authentic, Ms. Diaz talked in the spring interview about how she had come to recognize that she needed to move beyond focusing almost exclusively on student perseverance, something that had been a primary point of emphasis in other PD trainings she had attended over the years. She said, “[I want to learn about other practices], not just be stuck like we talked about on, you know, perseverance.” As the year progressed, more intentional conversations with Ms. Diaz also took place about the language practices found in the DMP. In particular, we discussed developing students’ mathematics register; the disciplinary-specific reading, writing, listening and speaking norms in mathematics that often differ from what is found in everyday English (Halliday, 1978). Ms. Diaz liked this focus on language because she had not particularly focused on the language of mathematics in the past. By the spring, mathematics language was an important focal point for Ms. Diaz, materializing as one of her orientations.

### **Supporting Ms. Baker: PD facilitator's response**

Analogous to Ms. Diaz, my conversations with Ms. Baker about how I could support her became more authentic as the year progressed. Also analogous to Ms. Diaz, Ms. Baker identified how she wanted more support with regard to learning how to implement the language practices found in the DMP. While Ms. Baker and I collaboratively developed lesson plans prior to problem-solving lessons, Ms. Baker would have liked more explicit support vis-à-vis language and how to encourage students to use the mathematics register at particular moments during lessons. Similar to Ms. Diaz, Ms. Baker articulated how she had minimal preparation with regard to supporting students to learn the mathematics register. Moreover, Ms. Baker shared that she would have liked to have had more supports given to her during the actual problem-solving lessons in which she was implementing the DMP. Ms. Baker shared that she would have liked to have talked more, to “huddle,” to cooperatively make instructional decisions “on the fly” (Stephan & Cobb, 2003) about DMP-based instructional moves that could have been made at particular moments during lessons. In the following, Ms. Baker described how she would have benefited from an active co-teaching model in which we collaboratively made instructional decisions:

I guess, thinking through a little more of when students are working through the problem. What might they say or what might they do? And how could I prompt them in the moment with that language? So, like when we’re walking around watching students work and listening to them share their ideas, what are some things I could be saying to them or asking them? I think this would have been helpful.

Similar to Ms. Diaz, mathematics language materialized as one of Ms. Baker’s orientations. Another orientation that emerged for Ms. Baker was her desire to engage in collaborative teaching and decision-making during the actual instruction of problem-solving lessons.

Reflecting on Ms. Baker’s Case: Ms. Baker’s case draws attention to challenges that can arise between teachers and a PD facilitator when the facilitator attempts to move into the role of instructional coach. Specifically, partly out of respect for a former student and mainly because of Ms. Baker’s competence, I tended to observe more during her lessons than engage in co-teaching with her. To a certain extent, Ms. Baker’s competence led me

to be complacent during problem-solving lessons; generally, I took on the role of a passive instructional coach who primarily engaged in observing and videotaping lessons rather than actively collaborating with Ms. Baker to co-teach lessons. The decision to largely take on a passive role when Ms. Baker taught was also a response to my declared process goal to collaborate with teachers in a respectful manner (i.e., in a manner that demonstrates that the PD facilitator does not have all the answers). Even though she had many areas of competence, Ms. Baker would have benefitted from a more proactive PD facilitator stance in which the facilitator actively collaborates with the teacher to co-teach problem-solving lessons. Furthermore, by taking on a relatively passive role during the lessons, I demonstrated a lack of support for Ms. Baker and her PD needs. That is, by taking the stance that Ms. Baker did not really need my assistance, I did not actualize an aspect of my identity concerning my belief that I have important information about the DMP to offer teachers to improve their instruction. Eventually, Ms. Baker shared her honest appraisal that I was not doing enough to help her learn how to use the DMP, particularly with regard to understanding how to support her students learn the mathematics register.

## Discussion

This study demonstrates the complexity of the decision-making process and the resultant practices of a PD facilitator to support two teachers to use an instructional protocol in mathematics. In Ms. Diaz' case, the decision was made to prioritize co-teaching as a means to support her professional development. From the outset of our collaboration with Ms. Diaz, our team observed that she was not implementing DMP instructional components during the problem-solving lessons. Given that one of my content goals was to support teachers directly in the classroom to implement these components, I decided together with team members to start co-teaching lessons with Ms. Diaz. This decision was made because we wanted to collaboratively model for Ms. Diaz how to implement components of the DMP. In addition, we wanted to model how instruction during problem-solving lessons could lead to student perseverance, while demonstrating how to support students to learn the mathematics register. These practices attended to Ms. Diaz' orientations, developing student perseverance and teaching the mathematics register. Thus, the decisions made about how to support Ms. Diaz' implementation of the DMP and the subsequent practices that were carried out were highly influenced by my process goal of wanting to support teachers directly in the classroom and by Ms. Diaz' orientations.

In the case of Ms. Baker, I initially took on relatively passive roles as an instructional coach during visits to her classroom when she implemented the DMP during problem-solving lessons. This was in response to my observations of her competencies relative to my content goal of teachers implementing the DMP's components. It was also a response to her abilities to prioritize student inquiry during instruction, one of my process goals. Essentially, my preliminary decision-making process about what practices to pursue as an instructional coach with Ms. Baker led to inaction because I did not know how I could help her. Nevertheless, through the development of a productive professional relationship with Ms. Baker, our relationship evolved. As the year progressed, she helped me to realize how I could support her by us "huddling" to make collaborative instructional decisions during the problem-solving lessons and through co-teaching to support her employ the DMP's language practices. These practices attended to Ms. Baker's orientations that involved engaging in collaborative teaching and promoting her students' learning of and use of the

mathematics register. Thus, the initial decision made about how to support Ms. Baker's implementation of the DMP was derived from my process goal of wanting to collaborate with teachers in a manner that demonstrates that the PD facilitator does not have all the answers. However, as our work progressed, my decision-making and subsequent practices were largely influenced by her two orientations; her aspiration to engage in collaborative teaching and decision-making during instruction and her desire to learn how to promote her students' use of mathematical language.

Looking across the two cases, prioritizing professional relationships played a pivotal role in the decision-making process and my subsequent practices. As productive professional relationships were cultivated with the participants through the year, forthright conversations became more common in which both teachers articulated what they perceived as the shortcomings of the PD supports they were being provided and what supports they actually wanted and needed. For example, Ms. Baker described how I could have supported her better by giving her prompts that used the mathematics register in response to her students' ideas. Ultimately, Ms. Diaz expressed frustration with the inadequate supports that were provided to her to implement the DMP. She indicated that she would have liked to have been given a list that detailed what was expected of her vis-à-vis employing the DMP during problem-solving lessons. It took time before Ms. Diaz was willing to explicitly articulate her PD needs. In the case of Ms. Baker, I came to understand how I needed to be more proactive with regard to both ascertaining and then attending to her PD needs. The development of our professional relationship led to Ms. Baker feeling comfortable enough to share how her PD needs were not being met and to me being vulnerable enough to realize that the passive role I had assumed as Ms. Baker's instructional coach was not serving her particularly well.

This study provides insights about the value of cultivating productive professional relationships with PD participants through deliberately working to solicit and accurately identify their needs (Noddings, 2002). Building productive professional relationships involves identifying with and humanizing teachers (Kitchen, 2018) through nurturing caring relationships that lead to trust (Battay & Chan, 2010; Sztajn et al., 2007). The decisions made and the subsequent practices were influenced from the beginning by my identification with the teachers because of our long-term relationships and the teachers' passion to make a difference in the lives of their minoritized students in particular. In both cases, identifying with the teachers from the outset of our collaborative work together played a crucial role in fostering productive professional relationships that led to honest communication among the participants and myself. Ultimately, cultivating productive professional relationships resulted in me having a better understanding of the participants' PD needs. In Ms. Diaz' case, the uncomfortable exchange that occurred at the conclusion of the fourth lesson served as a catalyst for Ms. Diaz frankly sharing what she believed was lacking in the PD that she was being offered. For Ms. Baker, our communication became more authentic as the year progressed and her actual PD needs were gradually revealed. Though cultivating productive professional relationships with teachers is time-consuming and challenging, this study demonstrates that honoring and sustaining teachers through the development of productive professional relationships can be a worthwhile undertaking. It was not enough to simply listen to teachers to understand their actual PD needs, our relationships had to evolve to a point where difficult conversations could take place, which resulted in the teachers trusting me enough to be sufficiently vulnerable to express their true PD needs.

The study also demonstrates potential challenges associated with PD facilitators strongly identifying with their participants when engaging with them as an instructional coach. In Ms. Diaz' case, I was strongly influenced by the fact that she was a veteran teacher who

was well-prepared to engage her students in demanding problem-solving lessons. Because of her strong identity and the many resources that she brought to the classroom, I failed to anticipate that she would not implement the components of the DMP during the problem-solving lessons. Ultimately, as authentic conversations emerged, we learned that Ms. Diaz would have liked to have had more explicit guidance about the DMP and how to implement its components. In the case of Ms. Baker, while my initial evaluation of her instruction was not necessarily faulty, it did not lead to supporting her actual PD needs. Because Ms. Baker was quite capable of implementing problem-solving (one of her resources) with her minoritized students using equitable teaching strategies (one of her identities), I struggled to recognize how to support her in the classroom. An additional challenge that emerged in both cases was that the teachers were tentative to share how the team (Ms. Diaz' case) and how I (Ms. Baker's case) could better support them during the problem-solving lessons. It took time to discover what the teachers actually required with regard to learning about and implementing the DMP. The reasons for this could include any of the following: (1) collaborating with outsiders is something teachers rarely do, particularly with people from higher education, (2) teachers may be hesitant or even resistant to providing honest feedback to outside collaborators because of a lack of trust, and (3) teachers may not know what their specific PD needs are, particularly when it comes to implementing a complex instructional protocol such as the DMP that they have never used before (Kitchen, 2023).

Knowing how to accurately attend to teachers' PD needs is a significant undertaking, particularly when PD facilitators work collaboratively with participants as instructional coaches in their classrooms. This study's findings point to the value of teachers having opportunities to evaluate the PD supports they are offered; their insights led me to adjust my decisions and practices as my work as an instructional coach progressed over the year. As the two teachers' PD needs became more apparent, I pursued practices to attempt to meet their individual needs. Importantly, the practices that were pursued with each teacher were not identical and were derived over a relatively significant period of time. The study's findings also hint at the potential value of engaging with participants in co-teaching to cultivate productive professional relationships. Future research could explicitly examine whether co-teaching with PD participants in their classrooms supports or hinders the development of productive professional relationships. Considering how the teachers, particularly Ms. Baker, were making suggestions near the end of the year about how co-teaching could benefit them, research could be undertaken to study how co-teaching can be leveraged to build trust with teachers. Differing co-teaching formats could be studied for their efficacy to encourage productive professional relationships as well.

Similar to Coles (2019), Karsenty et al. (2023), and Kuzle and Biehler (2015), this study brings to light the decision-making process and practices of a PD facilitator. A unique contribution of this study is that it is shown how my decision-making process and practices were carried out by a PD facilitator working in the role of an instructional coach. Another unique contribution of this study is that it is shown that a PD facilitator can cultivate honest and caring productive professional relationships with PD participants through identifying with them. It also proved essential that the participants saw that the team in Ms. Diaz' case, and me in Ms. Baker's case, were willing to be vulnerable ourselves; we were not always sure how to assist the teachers during the problem-solving lessons. As our vulnerabilities became more visible to the participants, our respective relationships became more authentic. Being professionally vulnerable with those we serve as PD facilitators shows our desire as facilitators to nurture social bonds with teachers as a means to advance interpersonal relationships (Aspelin, 2021; Lasky, 2005). Through being vulnerable with the participants

as a PD facilitator, both Ms. Diaz and Ms. Baker became more vulnerable and willing to reveal their actual PD needs.

In summary, my goal to support teachers' use of the DMP and my identification with the teachers played prominent roles in my decision-making and the practices that followed. Moreover, my decisions and practices were largely influenced by the PD participants' orientations and identities, their orientations in particular. Some practices pursued with both teachers were identical, such as supporting them to learn about and use the mathematics register (orientations for both teachers). However, there were differences in practices undertaken too, such as modeling student perseverance for Ms. Diaz (one of her orientations) and huddling with Ms. Baker during instruction (one of her orientations). More research is needed on PD facilitators' decision-making processes and the resultant practices they pursue with their PD participants (Coles, 2019; Karsenty et al., 2023; Kuzle & Biehler, 2015). More research is also needed to examine how the cultivation of productive professional relationships can be scaled up for use with more than two teachers. In addition to the fact that there were only two PD participants, a limitation of this study is that I had worked with both participants previously. A future study could be carried out with teachers with whom the researcher has no past relationships with to examine the benefits of fostering productive professional relationships with them to support their PD needs.

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## References

- Aspelin, J. (2021). Teaching as a way of bonding: A contribution to the relational theory of teaching. *Education Philosophy and Theory*, 53(6), 588–596. <https://doi.org/10.1080/00131857.2020.1798758>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59, 389–407.
- Bartell, T. G. (2011). Caring, race, culture, and power: A research synthesis toward supporting mathematics teachers in caring with awareness. *Journal of Urban Mathematics Education*, 4(1), 50–74. <https://doi.org/10.21423/jume-v4i1a128>
- Battey, D., & Chan, A. (2010). Building community and relationships that support critical conversations on race: The case of Cognitively Guided Instruction. In Mary Q. Foote (Ed.), *Mathematics teaching and learning in K-12* (pp. 137–149). New York: Palgrave Macmillan US. [https://doi.org/10.1057/9780230109889\\_10](https://doi.org/10.1057/9780230109889_10)
- Beswick, K., & Goos, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *Journal of Mathematics Teacher Education*, 21, 417–427. <https://doi.org/10.1007/s10857-018-9416-4>
- Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, 33(8), 3–15.
- Borko, H., Koellner, K., & Jacobs, J. (2014). Examining novice teacher leaders' facilitation of professional development. *Journal of Mathematical Behavior*, 33, 149–167.
- Borko, H., Liston, D., & Whitcomb, J. (2007). Genres of empirical research in teacher education. *Journal of Teacher Education*, 58, 3–11. <https://doi.org/10.1177/0022487106296220>

- Borowski, R. S., & Rupe, K. M. (2023). Can we figure this out together? Surfacing beliefs through collaborative planning. *Philosophy of Mathematics Education Journal*, 40, 220.
- Campbell, P. F., & Malkus, N. N. (2011). The impact of elementary mathematics coaches on student achievement. *Elementary School Journal*, 111(3), 430–454.
- Chval, K. B., Arbaugh, F., Lannin, J. K., van Garderen, D., Cummings, L., & Huey, M. E. (2010). The transition from experienced teacher to mathematics coach: Establishing a new identity. *The Elementary School Journal*, 111(1), 191–216. <https://doi.org/10.1086/653475>
- Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal for Research in Mathematics Education*, 40(1), 40–68.
- Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of mathematics teaching at scale. *Mathematics Teacher Education and Development*, 13(1), 6–33.
- Coles, A. (2019). Facilitating the use of video with teachers of mathematics: Learning from staying with the detail. *International Journal of STEM Education*, 6(1), 1–12. <https://doi.org/10.1186/s40594-018-0155-y>
- Copur-Gencturk, Y., & Papkonstantinou, A. (2016). Sustainable changes in teacher practices: A longitudinal analysis of the classroom practices of high school mathematics teachers. *Journal of Mathematics Teacher Education*, 19, 575–594. <https://doi.org/10.1007/s10857-015-9310-2>
- Elliott, R., Kazemi, E., Lesseig, K., Mumme, J., Carroll, C., & Kelley-Petersen, M. (2009). Conceptualizing the work of leading mathematical tasks in professional development. *Journal of Teacher Education*, 60(4), 364–379. <https://doi.org/10.1177/0022487109341150>
- Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM – Mathematics Education*, 53(2), 245–262. <https://doi.org/10.1007/s11858-020-01213-2>
- Even, R. (2005). Integrating knowledge and practice at MANOR in the development of providers of professional development for teachers. *Journal of Mathematics Teacher Education*, 8(4), 343–357. <https://doi.org/10.1007/s10857-005-0855-3>
- Even, R., Robinson, N., & Carmeli, M. (2003). The work of providers of professional development for teachers of mathematics: Two case studies of experienced practitioners. *International Journal of Science and Mathematics Education*, 1(2), 227–249.
- Fishman, J. J., Marx, R. W., Best, S., & Tal, R. T. (2003). Linking teacher and student learning to improve professional development in systemic reform. *Teaching and Teacher Education*, 19(6), 643–658. [https://doi.org/10.1016/S0742-051X\(03\)00059-3](https://doi.org/10.1016/S0742-051X(03)00059-3)
- Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, 38(4), 915–945.
- Halliday, M. A. K. (1978). *Language as social semiotic*. Edward Arnold.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371–406. <https://doi.org/10.3102/00028312042002371>
- Karsenty, R., Pöhler, B., Schwartz, G., & Prediger, S. (2023). Processes of decision-making by mathematics PD facilitators: The role of resources, orientations, goals, and identities. *Journal of Mathematics Teacher Education*, 26, 27–51. <https://doi.org/10.1007/s10857-021-09518-z>
- Kastberg, S., & Kitchen, R. (2024). Relational practices in mathematics teacher education: Theorizing a construct to inform our practice. B. Benken (Ed.), *Reflection on Past, Present and Future: Paving the Way for the Future of Mathematics Teacher Education* (pp. 83–100). Association of Mathematics Teacher Educators Professional Book Series, Volume 5.
- Kastberg, S., Lischka, A. E., & Hillman, S. L. (2020). Characterizing mathematics teachers' written feedback to prospective teachers. *Journal of Mathematics Teacher Education*, 23, 131–152. <https://doi.org/10.1007/s10857-018-9414-6>
- Kennedy, M. M. (2016). How does professional development improve teaching? *Review of Educational Research*, 86(4), 945–980. <https://doi.org/10.3102/0034654315626800>
- Kitchen, R. (2023). Teachers talk about implementing problem-solving based instruction at a highly diverse elementary school. In: *Proceedings of the Thirteenth Conference of the European Society for Research in Mathematics Education (CERME 13, July 10–14, 2023)*.
- Kitchen, R. S. (2018). A commentary with urgency: Looking across theoretical perspectives to put relationship building with underserved students at the forefront of our work. In S. E. Kastberg, A. M. Tyminski, A. E. Lischka, & W. B. Sanchez (Eds.), *Building support for scholarly practices in*



- mathematics methods* (pp. 343–358). Association of Mathematics Teacher Educators and Information Age Publishing.
- Kitchen, R. (2022). The power of their ideas: Leveraging teachers' mathematical ideas in professional development. *International Journal of Mathematical Education in Science and Technology*, 53(7), 1835–1858. <https://doi.org/10.1080/0020739X.2020.1847337>
- Kitchen, R., Castellón, L. B., Matute, K., & Mosqueda, E. (2024). Examining the interplay of design principles in an instructional intervention for multilingual learners: The discursive mathematics protocol. *International Journal of Mathematical Education in Science and Technology*. <https://doi.org/10.1080/0020739X.2024.2391967>
- Kitchen, R., Martínez-Archuleta, M., Gonzales, L., & Bicer, A. (2021). Actualizing change after experiencing significant mathematics PD: Hearing from teachers of colour about their practice and mathematical identities. *Education Sciences*, 11(11), Article 710726. <https://doi.org/10.3390/educsci11110710>
- Knight, J. (2007). *Instructional coaching: A partnership approach to improving instruction*. Corwin Press.
- Kraft, M. A., & Blazar, D. (2018). Taking teacher coaching to scale: Can personalized training become standard practice? *Education next*, 18(4), 68–74.
- Kuzle, Ana, & Biehler, Rolf. (2015). Examining mathematics mentor teachers' practices in professional development courses on teaching data analysis: Implications for mentor teachers' programs. *ZDM Mathematics Education*, 47(1), 39–51. <https://doi.org/10.1007/s11858-014-0663-2>
- Lasky, S. (2005). A sociocultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education*, 21(8), 899–916. <https://doi.org/10.1016/j.tate.2005.06.003>
- Lischka, A. E., Gerstenschlager, N., & Webster, J. (2023). Dismantling traditional structures of assessment practice: Self-study as a path to understanding alternative assessment systems. *Philosophy of Mathematics Education Journal*, 40, 145.
- Liu, S., & Phelps, G. (2020). Does teacher learning last? Understanding how much teachers retain their knowledge after professional development. *Journal of Teacher Education*, 71(5), 537–550. <https://doi.org/10.1177/0022487119886290>
- Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics* (3rd ed). Corwin Press.
- Loughran, J. (2007). Researching teacher education practices: Responding to the challenges, demands and expectations of self-study. *Journal of Teacher Education*, 58, 12–20. <https://doi.org/10.1177/0022487106296217>
- Loughran, J. J., & Northfield, J. (1998). A framework for the development of self-study practice. In M. L. Hamilton (Ed.), *Reconceptualizing teaching practice: Self-study in teacher education* (pp. 7–18). Routledge.
- Lunenberg, M., Zwart, R., & Korthagen, F. (2010). Critical issues in supporting self-study. *Teaching and Teacher Education*, 26(6), 1280–1289. <https://doi.org/10.1016/j.tate.2009.11.007>
- Majority minority in the United States. (n.d.). [https://en.wikipedia.org/wiki/Majority\\_minority\\_in\\_the\\_United\\_States](https://en.wikipedia.org/wiki/Majority_minority_in_the_United_States)
- Miles, M. B., Huberman, A. M., & Saldaña, J. (2013). *Qualitative data analysis: A methods sourcebook*. Sage Publications.
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.
- National Center for Education Statistics. (2018). *Percentage distribution of public school teachers, by race/ethnicity and state: 2017–18*. [https://nces.ed.gov/surveys/ntps/tables/ntps1718\\_ftable01\\_t1s.asp](https://nces.ed.gov/surveys/ntps/tables/ntps1718_ftable01_t1s.asp)
- National Center for Education Statistics. (2023). *Characteristics of public school teachers*. <https://nces.ed.gov/programs/coe/indicator/clr/public-school-teachers>
- Noddings, N. (2002). *Educating moral people: A caring alternative to character education*. Teachers College Press.
- Pinnegar, S., Hutchinson, D. A., & Hamilton, M. L. (2020). Role of positioning, identity, and stance in becoming S-STEP researchers. In J. Kitchen, A. Berry, S. M. Bullock, A. R. Crowe, M. Taylor, H. Guðjónsdóttir, & L. Thomas (Eds.), *International handbook of self-study of teaching and teacher education practices* (pp. 97–133). Singapore: Springer Singapore. [https://doi.org/10.1007/978-981-13-6880-6\\_4](https://doi.org/10.1007/978-981-13-6880-6_4)
- Polly, D., Algozzine, R., Martin, C. S., & Mraz, M. (2015). Perceptions of the roles and responsibilities of elementary school mathematics coaches. *International Journal of Mentoring and Coaching in Education*, 4(2), 126–141. <https://doi.org/10.1108/IJMC-E-2014-0030>
- Pólya, G. (1986). *How to solve it: A new aspect of mathematical method*. Princeton University Press. (Original work published in 1945)
- Riessman, C. K. (1993). *Narrative analysis*. Sage Publications Inc.



- Rodriguez, A. J., & Kitchen, R. S. (Eds.). (2005). *Preparing mathematics and science teachers for diverse classrooms: Promising strategies for transformative pedagogy*. Lawrence Erlbaum Associates.
- Rösken-Winter, B., Schüler, S., Stahnke, R., & Blömeke, S. (2015). Effective CPD on a large scale: Examining the development of multipliers. *ZDM*, 47(1), 13–25. <https://doi.org/10.1007/s11858-014-0644-5>
- Schoenfeld, A. (1985). Metacognitive and epistemological issues in mathematical understanding. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 361–380). Erlbaum.
- Schoenfeld, A. H. (2010). *How we think: A theory of goal-oriented decision making and its educational applications*. Routledge.
- Shulman, L. S. (2004). *The wisdom of practice: Essays on teaching, learning, and learning to teach*. Jossey-Bass.
- Silver, E. A., & Smith, M. S. (1996). Building discourse communities in mathematics classrooms: A worthwhile but challenging journey. In P. Elliott (Ed.), *Communication in mathematics, K-12 and beyond* (pp. 20–28) [1996 yearbook of the National Council of Teachers of Mathematics]. National Council of Teachers of Mathematics.
- Sowder, J. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 157–224). Charlotte: Information Age.
- Stephan, M., & Cobb, P. (2003). The methodological approach to classroom-based research. In N. Pateman (Ed.), *Supporting students' development of measuring conceptions: Analyzing students' learning in social context* (Vol. 12). Reston, VA: National Council of Teachers of Mathematics.
- Sztajn, P., Hackenberg, A. J., White, D. Y., & Alexsaht-Snyder, M. (2007). Mathematics professional development for elementary teachers: Building trust within a school-based mathematics education community. *Teaching and Teacher Education: An International Journal of Research and Studies*, 23(6), 970–984. <https://doi.org/10.1016/j.tate.2006.04.023>
- Tzur, R. (2001). Becoming a mathematics teacher-educator: Conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education*, 4, 259–283.
- United States Census Bureau. (2022). *QuickFacts: New Mexico*. <https://www.census.gov/quickfacts/NM>
- Vanassche, E., & Kelchtermans, G. (2016). Facilitating self-study of teacher education practices: Toward a pedagogy of teacher educator professional development. *Professional Development in Education*, 42(1), 100–122. <https://doi.org/10.1080/19415257.2014.986813>
- Webb, N. M., Franke, M. L., Johnson, N. C., Ing, M., & Zimmerman, J. (2021). Learning through explaining and engaging with others' mathematical ideas. *Mathematical Thinking and Learning*. <https://doi.org/10.1080/10986065.2021.1990744>
- Zaslavsky, O., & Leikin, R. (2004). Professional development of mathematics teacher educators: Growth through practice. *Journal of Mathematics Teacher Education*, 7(1), 5–32.

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